Math 10A with Professor Stankova
Worksheet, Discussion #15; Friday, 9/29/2017
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Anti-Derivatives

Example

1. Find an antiderivative of $\dfrac{1}{2x}$.

> **Solution:** By the constant integration law, we know that an antiderivative of $\dfrac{1}{2x}$ is half an antiderivative of $\dfrac{1}{x}$, thus it suffices to find an antiderivative of $\dfrac{1}{x} = x^{-1}$. Ideally, we would take $\dfrac{x^{n+1}}{n+1}$ for an antiderivative of x^n, but $-1+1=0$ so we cannot use the power antiderivative rule. But, we remember that the derivative of $\ln x$ is $\frac{1}{x}$. So, an antiderivative of $\dfrac{1}{2x}$ is $\dfrac{\ln x}{2}$.

Problems

2. Find an antiderivative of $5e^x$.

> **Solution:** An antiderivative of e^x is e^x and so by the constant integration law, an antiderivative of $5e^x$ is $5e^x$.

3. Find an antiderivative of $x + \sqrt{x}$.

> **Solution:** An antiderivative of x is $\frac{x^2}{2} + 5$ and an antiderivative of $\sqrt{x} = x^{1/2}$ is $\frac{2}{3}x^{3/2} + 10$ and so by the addition integration law, an antiderivative of $x + \sqrt{x}$ is $\frac{x^2}{2} + \frac{2}{3}x^{3/2} + 15$.

4. Find an antiderivative to $8t^3 + 15t^2$.

Solution: An antiderivative of t^3 is $\frac{t^4}{4}$ and an antiderivative of t^2 is $\frac{t^3}{3} + \pi$. So using the constant and addition antiderivative law, we get that an antiderivative of $8t^3 + 15t^2$ is $8\frac{t^4}{4} + 15\frac{t^3}{3} + 15\pi = 2t^4 + 5t^3 + 15\pi$.

5. Find an antiderivative to e.

Solution: We can write $e = e \cdot x^0$. An antiderivative to x^0 is $x+1$ so by the constant antiderivative law, an antiderivative to e is $e(x + 1) = ex + e$.

6. Find an antiderivative to $\cos u$.

Solution: One choice is $\sin u + 5$.

7. Find an antiderivative to $\sin(2t)$.

Solution: We want to guess $-\cos(2t)$ but the derivative of $-\cos(2t)$ is $\sin(2t) \cdot 2$ by the chain rule. So we can multiply by one half to get a function that works. So one choice is $\frac{-\cos(2t)}{2}$.